

The Monochord in the Medieval and Modern Classrooms

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For Cecil Adkins

Throughout the Middle Ages, the monochord was a vehicle for turning abstract number ratios into empirical evidence perceptible by eye and ear, making musical concepts physically present for the student. A section on how to divide the monochord was a standard inclusion in the music theorist's treatise, and about 150 monochord divisions survive from the ninth through the fifteenth century.¹ Sometimes these discussions seem to be inserted into the medieval texts merely as an expected component, but in other treatises they are accompanied by instructions or insights suggesting that the theorist had a specific lesson plan in mind. Authors of the period had different pedagogical aims in the use of the monochord, which ranged from the pragmatic need to teach pitch and interval recognition, to the academic need for understanding the derivation of notes by ratio, on to the more esoteric realm of understanding the complex ratios that were at the heart of tuning and temperament.

The physicality and the presence of the monochord can play an important role in the modern music history classroom. Indeed, one of the challenges of teaching the music of the Middle Ages is a lack of presence—trying to talk about things that aren't fully there. This simple fact is highlighted in Richard Taruskin's decision to begin his five-volume *Oxford History of Western Music* with "Music from the Earliest Notations."² And in the condensed version of that series, designed for undergraduate teaching, the chapter subheadings highlight this ambiguity: "Historical *Imagination*," "Christian Beginnings, as

1. These divisions can be found in Christian Meyer, *Mensura monochordi: La division du monochorde, IX^e-XV^e siècles* (Paris: Publications de la Société française de musicologie, 1996).

2. Richard Taruskin, *Music from the Earliest Notations to the Sixteenth Century*, Volume 1 of *The Oxford History of Western Music* (Oxford: Oxford University Press, 2005).

Far as We Know Them,” and “The Legend of St. Gregory” (authors’ emphasis).³ A half-millennium of chant can only be conjectured upon for its lack of notation.

Matters don’t markedly improve even when we have written music. For pragmatic reasons, we primarily teach from modern editions and recordings and the students struggle to untangle the musical text from its edition or performance. If they hear an added drone or instrumental doubling, or even the rhythmic realization of a verse’s implied meter, they view it as an integral part of the piece itself, and no amount of reminders can fully shake them of this view.

All of this is compounded by the central role of memory in the learning, the performance, and the composition of music.⁴ Young clerics memorized entire bodies of chant. Students of early counterpoint took instruction set to verse and etched it in their memory along with enormous numbers of contrapuntal formulas and useful melismas, to the point that when we show students a piece of early polyphony we can only hint at the multitude of other realizations that once existed. In short, much of what we deal with is absent, highly mediated, or a stand-in for a larger, lost body of music.

To counteract this, material objects can play an important role in our classrooms, and these objects can provide critical insight for our students. For example, facsimile editions can teach a great deal merely by their physicality. Students may learn a great deal from viewing facsimile images of Notre Dame polyphony in their text, but until they can see (and hold) a page in its actual size, they will not realize how different it is from the “choral score” they are used to. From this, they can intuit that these are pieces sung largely from memory and only preserved in the small manuscript for the purpose of learning.⁵ But for the most part, these remain objects, and seldom become agents—that is, objects that allow students to partially enter the world that they are studying. If, on the other hand, we could have the students sing examples from facsimile editions—struggling with the process of notation and rhythmic coordination—they would gain an experiential knowledge of the material, knowledge that can serve to lock in specific concepts by pairing them with concrete experience. But this approach takes advanced skills and far more

3. Richard Taruskin and Christopher H. Gibbs, *The Oxford History of Western Music*, college edition (New York: Oxford University Press, 2013).

4. The elements outlined here are covered in detail in Anna Maria Busse Berger, *Medieval Music and the Art of Memory* (Berkeley: University of California Press, 2005).

5. The smallest of these *Magnus liber organi* sources (the Madrid manuscript) is a mere 16.5 cm x 11.5 cm—smaller than a paperback novel. Available facsimiles of the primary Notre Dame sources reproduce this size, though we can’t assume this in all facsimile editions we might use. Lacking access to a facsimile edition, it is easy enough to make a photocopy scaled to size for the students to examine.

time than most instructors can provide in the classroom. There are, however, some objects that can quickly and easily provide the concrete involvement that leads to an immediate and deeper understanding. One of these is the monochord.

Teaching musical ratios with the monochord allows the student, medieval or modern, to explore the physical representation of the abstract numerical concepts. In the modern classroom, using the monochord has another important purpose—it allows modern students to share experiences with the medieval students. For the time that they are in the classroom, they are not only learning about the concept, but are also experiencing the pedagogical world of the medieval classroom, bringing a deeper reality to an otherwise abstract series of concepts.

Our essay comprises three parts. The first section provides an overview of how the monochord was used by medieval theorists. While many of the sources cited will be familiar to readers from the source readings of Weiss and Taruskin or Strunk, this summary will highlight the monochord's use as a pedagogical instrument and introduce the reader to the techniques the authors use in class. The second section applies these concepts and techniques in a single course module in which the students will follow Guido's instructions to create a Pythagorean division of the monochord.⁶ The final section suggests ways in which students and teachers can go beyond the basic division of the monochord to explore other concepts.

I. The Monochord in the Medieval Classroom: Sources and Pedagogy

As with so many other aspects of early music theory, Boethius was responsible for preserving Greek divisions of the monochord for his medieval followers. He provided three systems: one of placing a bridge at a specified place beneath a string to compare the sounds of the two lengths of string; a second of dividing the monochord by length to demonstrate the three genera of tetrachords (which starts with the somewhat impractical division of the string into 9,216 equal units); and a third of manually dividing the string by ratios to

6. Most of the divisions we will discuss, as well as most that survive in the sources, result in identical Pythagorean tunings. Pythagorean tuning was the predominant tuning system used throughout the Middle Ages. Attributed to the Greek philosopher Pythagoras (6th century BCE), the system is based on simple ratios using the numbers 1, 2, 3, and 4 to create consonant intervals. The system has pure unisons (1:1), octaves, (2:1), fifths (3:2), and fourths (4:3), as well as ratios that can be derived from these numbers, such as the pure major second (9:8). However, major thirds are not pure (5:4), but made up of two pure major seconds (9:8 + 9:8, or 81:64).

create a scale (to use modern terminology). This final, manual division was overwhelmingly adopted by medieval theorists.⁷

One of the earliest post-Boethian descriptions of the monochord is in the *Dialogus de musica* (10th century) by the so-called Pseudo-Odo.⁸ He provides a physical description of the instrument as well as instructions on how to use it. For Pseudo-Odo, the monochord was fundamental to his teaching of music:

(Disciple) What is music?

(Master) The science of singing truly and the easy road to perfection in singing.

(D) How so?

(M) As the teacher first shows you all the letters on a slate, so the musician introduces all the sounds of melody on the monochord.

(D) What is the monochord?

(M) It is a long rectangular wooden chest, hollow within like a cithara; upon it is mounted a string, by the sounding of which you easily understand the varieties of sounds.

(D) How is the string itself mounted?

(M) A straight line is drawn down the middle of the chest, lengthwise, and points are marked on the line at a distance of one inch from each end. In the spaces outside these points two end-pieces are set, which hold the string so suspended above the line that the line beneath the string is of the same length as the string between the two end-pieces.

(D) How does one string produce many different sounds?

(M) The letters, or notes, used by musicians are placed in order on the line beneath the string, and when the bridge is moved between the line and the string, shortening or lengthening it, the string marvelously reproduces any chant by means of these letters.⁹

So that his student may use the monochord to sound out chant, the teacher proceeds to give instructions on how to mark the gamut on the monochord. This is achieved through a process of dividing the string into equal segments. As his instructions are clear and easy to follow, it is a simple matter to reproduce his method using a piece of paper two or three feet long (a piece of receipt paper works well), and a set of dividers or compass.

7. For a more detailed discussion of Boethius's divisions, see Cecil Adkins, "The Theory and Practice of the Monochord" (PhD dissertation, University of Iowa, 1963), 95–105, and Meyer, xxvi–xxix.

8. A transcription of the *Dialogus* can be found in *Scriptores ecclesiastici de musica sacra potissimum*, 3 vols., ed. Martin Gerbert (St. Blaisen: Typis San-Blasianus, 1784; reprint ed., Hildesheim: Olms, 1963), 1:265–84. A new edition is currently being prepared by Karl-Werner Gumpel. A translation of portions of the work is found in Oliver Strunk, *Source Readings in Music History*, revised edition, Leo Treitler, general editor (New York, W. W. Norton, 1998), 198–210.

9. Strunk, 200–1.

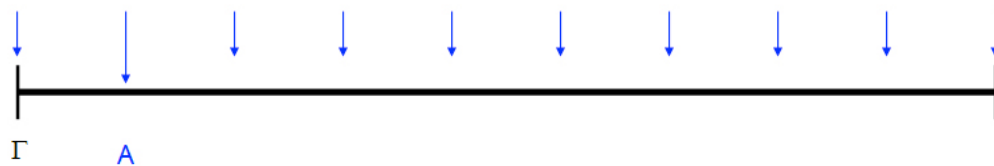
A straight line drawn down the center of the paper represents the string. The author begins:

At the first end-piece of the monochord . . . place the letter Γ , that is, a Greek G^{10} . . . Carefully divide the distance from Γ to the point placed at the other end into nine parts.¹¹

Starting at one end of the line, swing the dividers down the line, aiming to reach the other end in nine swings. Through trial and error (as the medieval student did) the modern student will arrive at the proper width. After finding the correct measurement, you can continue with Pseudo-Odo's instructions: "where the first ninth from Γ ends, write the letter A; we shall call this the first step."¹²

This creates a ratio between the length of the whole string and the length of the stopped string. The whole string is nine units long, and the stopped string is eight units long, making the ratio 9:8 (see **Figure 1**).

Figure 1: Pseudo-Odo's first division to find A.



To continue with Pseudo-Odo: "Then, similarly, divide the distance from the first letter, A, to the end into nine, and at the first ninth, place the letter B for the second step."¹³ Now you must shorten your dividers slightly to find the new measurement. The same ratio 9:8 is created by taking the shorter length of string, from A to the end, dividing it into nine units, and marking the next note at eight of those units (see **Figure 2**).

At this point two Pythagorean whole tones have been created by using a ratio 9:8. Pseudo-Odo follows with the semitone from B to C. Finding this half step in the same manner would involve creating a ratio of 256:243. While medieval theorists knew this ratio, the authors have found only one treatise that suggests measuring this interval.¹⁴ The more practical approach is to go

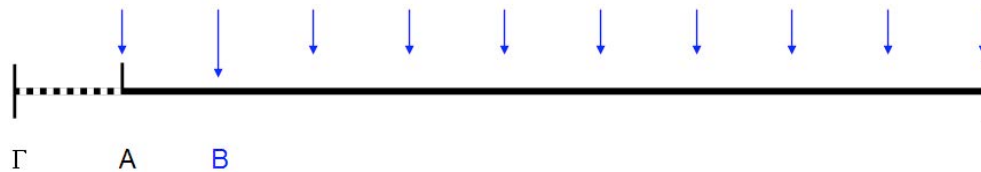
10. Here we are using Guido's pitch nomenclature: Γ A B C D E F G a b/b \flat c d e f g a' b'/b \flat ' c' d' e'.

11. Strunk, 201.

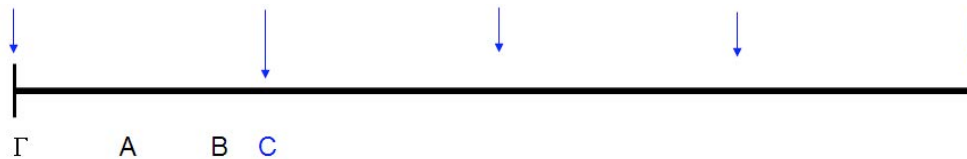
12. Ibid.

13. Ibid.

14. The procedure is found in an anonymous fifteenth-century manuscript, transcribed in Meyer, 132–33. It is done by dividing the distance from B to the end into two parts, and

Figure 2: Pseudo-Odo's second division to find B.

back to Γ and find C by the ratio of a fourth, 4:3, creating the complicated semitone ratio simply by leaving it as a remainder. This is just what Pseudo-Odo does: “Then return to the beginning, divide by four from Γ , and for the third step write the letter C” (see **Figure 3**).¹⁵ He continues by moving up by fourths from these initial notes: finding D from A, E from B, and F from C (see **Figure 4**). He concludes the gamut by finding $b\flat$ from F, and then dividing the string in half from each lower note to find the higher octave.

Figure 3: Pseudo-Odo's third division to find C.

The student of the dialogue, surveying his finished monochord, notices that even though the ratio 9:8 is only used to find the first two whole steps, every other whole step, say from C to D, is also a ratio of 9:8. The master agrees that there are multiple ways to find each note: for instance, D can be found as a 4:3 ratio from A, a 3:2 ratio from G, or a 9:8 ratio from C (see **Figure 5**). Because of this redundancy, identical Pythagorean tunings can be found in myriad ways, as we shall see. The choices made by the theorist about how to present his division can reveal his motives about how he intends the student to use the knowledge.

For Pseudo-Odo, the first intent was to introduce the gamut in an intuitive way, for he found the notes in ascending order. Once the student understands the gamut and has a marked monochord, he can use it to learn music: “When the boys mark some antiphon with these letters, they learn it better and

then continuing the process, bisecting each subsequent division (i.e., 2, 4, 8, 16, 32, 64, 128, 256), and then measuring off 13 of those segments to find C.

15. Strunk, 201. Standard dividers are likely too small to measure one quarter of the length. In such cases, you must convert to eighths, so that 8:6 would substitute for 4:3. This will be clearer when we discuss Guido's division below.

Figure 4: Pseudo-Odo's divisions to find D, E, and F.

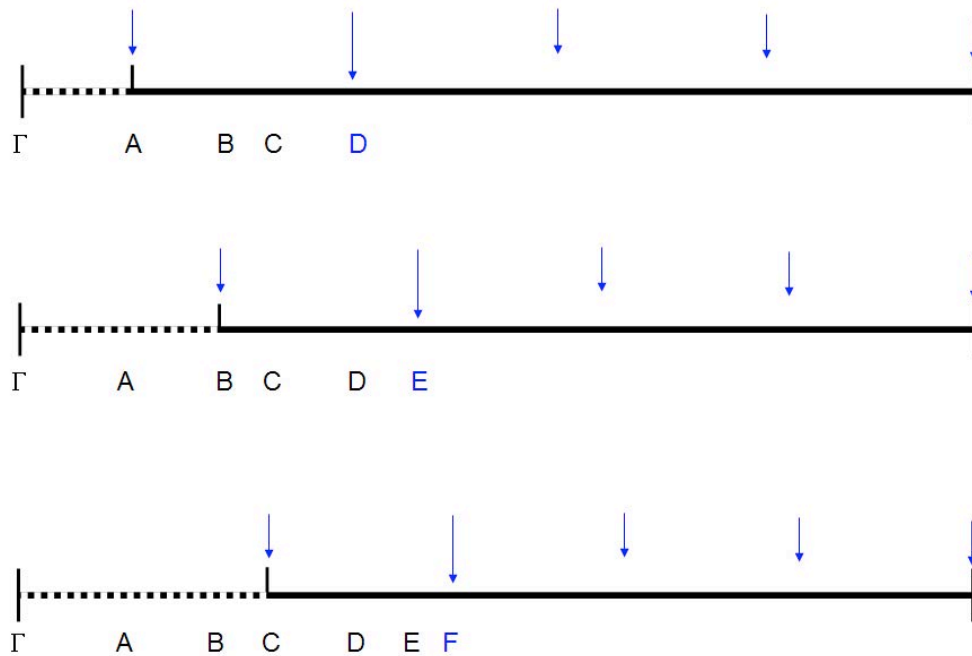
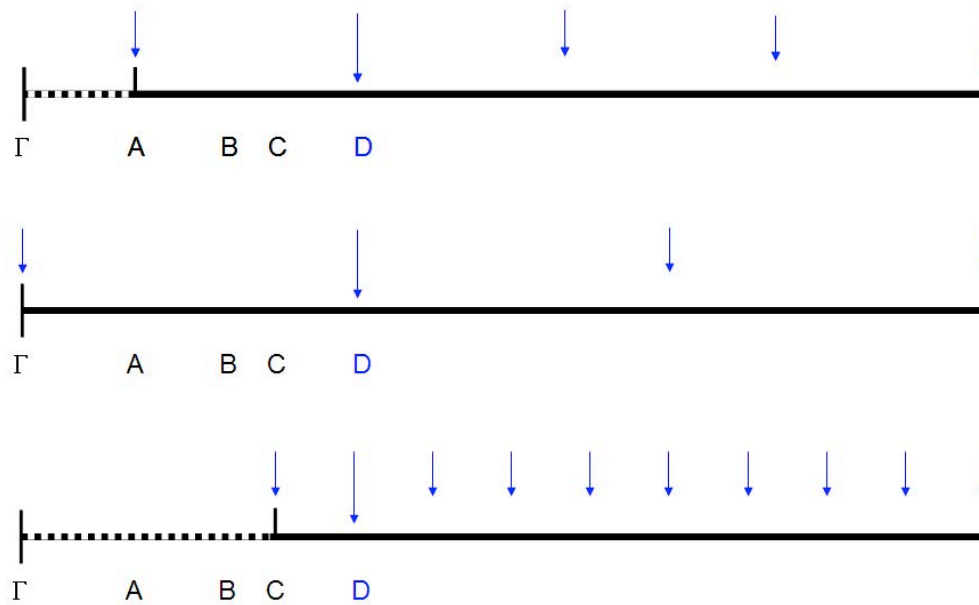


Figure 5: Redundancy in Pythagorean tuning.



more easily from the string than if they heard some one sing it; and they are able after a few months' training to discard the string and sing by sight alone, without hesitation, music that they have never heard."¹⁶ The rest of the *Dialogus* contains a discussion of consonances and modes, in a manner that Claude Palisca hails as "the clearest exposition so far" because he can refer to the notes of the gamut created on his monochord.¹⁷

Guido of Arezzo is the next theorist to contribute significantly to the use of the monochord. While his initial intent is much the same as Pseudo-Odo's, he applies the monochord to some of his new teaching techniques in order to help the student learn chant by hearing the pitches, and eventually leaves even that behind for more efficient methods. In his earliest surviving writing, the *Micrologus* (c. 1025), he provides two monochord divisions. The first is along the same lines as Pseudo-Odo's, wherein the notes are found in order with as few types of ratios as possible. His second tuning, however, is designed to be quicker, and is more pragmatic than didactic in its approach. It allows the student to complete the division rapidly in order that he might have more time to use the monochord as a pitch producing tool.

Guido begins his second division begins like this: "You make nine steps, that is [equal] segments, from Γ to the other end. The first step will end at A."¹⁸ Thus far, he is following his first division. But now, instead of resetting the dividers to find B from A, he finds more notes in the A and D pitch classes using this same setting. If A to the end is eight units long, then D can be found as 8:6, the octave a at 8:4, and so forth. To return to Guido: "The first step will end at A, the second will have no letter, the third will end at D, the fourth will be unlettered, the fifth will end at a, the sixth at d, the seventh at [a'], and the others will be unlettered" (see **Figure 6**, top line).¹⁹

Only now do you return to A, resetting your dividers into ninths, and finding all of notes in the pitches classes for B and E in a similar manner. Next, going back to Γ and dividing the whole string into four, you can find not only C, but the next two G's as well. Dividing C to the end into four yields F and two more C's, and finally dividing F to the end into four produces $b\flat$ and f. The gamut has been found using only five settings of the dividers (see

16. Strunk, 200–1.

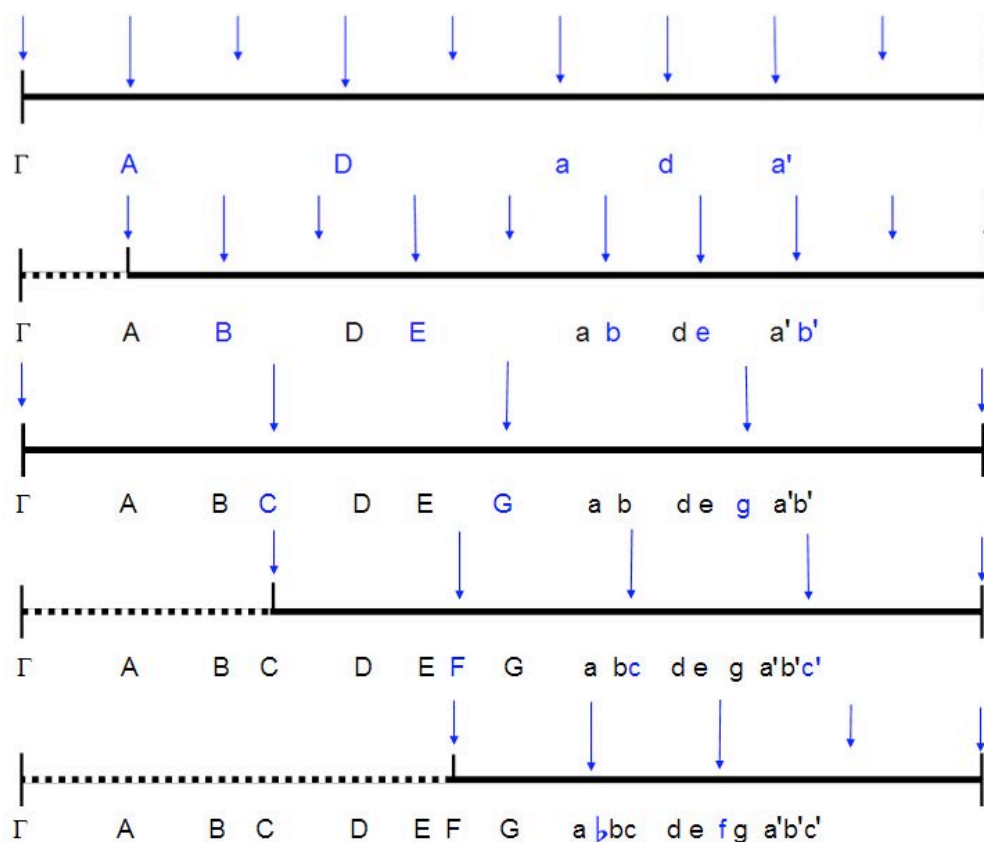
17. Claude V. Palisca and Ian D. Bent, "Theory, theorists, §5: Early Middle Ages," in *Grove Music Online. Oxford Music Online*, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/44944> (accessed February 22, 2012).

18. Warren Babb, *Hucbald, Guido, and John on Music: Three Medieval Treatises* (New Haven: Yale University Press, 1978), 60. Babb based his translation on Guido Aretinus. *Micrologus*, ed. Jos Smits van Waesberghe, *Corpus Scriptorum de Musica* 4 ([Rome]: American Institute of Musicology, 1955).

19. Babb, 60.

Figure 6).²⁰ However, it is not as intuitive as the previous division. Rather than finding the notes in ascending order with only a few kinds of ratios, this scheme locates them in seemingly random order across the entire gamut, using a constantly changing series of ratios. But at the same time, following the process provides a satisfying sense of completion as the last missing letters are filled in.

Figure 6: Guido's five-step system.



For Guido, this would be a practitioner's monochord. This division is for someone who knows all about the gamut and the ratios (or has no interest in that aspect), and wants simply to quickly create a marked monochord so he can use it to produce pitches. Guido presents multiple applications for the monochord. Later in the *Micrologus*, he proposes a precursor to his

20. Guido does not include e' in the *Micrologus*, and bb' and d' are lacking from this division. In different sources for the *Micrologus*, Guido handles the lack of bb' and d' differently—providing a quick way of finding one or both, or ignoring the issue altogether as he does in the sources used in van Waesberghe's edition.

solmization system, and gives directions on how to use it with a monochord: “Let us take these five vowels. Perhaps, because they bring such euphony to words, they will offer no less harmony to the neumes. Let them be placed in succession beneath the letters of the monochord, and since they are only five, let them be repeated until beneath each note its particular vowel is written.”²¹ Now students can make the connection between pitch and vowel sound, or later the solmization, for that syllable.

Guido also applies the monochord to his staff notation:

And in order that you may understand to which lines or spaces each sound belongs, certain letters of the monochord are written at the beginning of the lines or spaces. And the lines are also gone over in colors . . . We use two colors, namely yellow and red, and by means of them I teach you a very useful rule that will enable you to know readily to what tone and to what letter of the monochord every neume and any sound belongs; that is, if—as is greatly convenient—you make frequent use of the monochord and of the formulas of the modes.²²

Thus Guido has provided students with a quick way to divide their monochord, and instructions on how to use it in sight singing and reading. However, he recognizes that students should not always be dependent on their monochord, and should eventually leave it behind as they continue with other more sophisticated tools for note reading and sight singing. In his later *Epistle*, in which he explains the use of his fully-developed solmization system, he advises: “You sound on the monochord the letters belonging to each neume, and by listening you will be able to learn the melody as if from hearing it sung by a teacher. But this procedure is childish, good indeed for beginners, but very bad for pupils who have made some progress.”²³ Instead, they should gain experiential knowledge of the relationships of the pitches to each other, thus enabling them to sing a new piece of chant in tune and in the correct mode.²⁴

Even though Guido had developed more efficient methods for learning chant, theorists continued to include monochord divisions in their writings. This suggests that the monochord was more than a pitch-producing machine; it was valued for its ability to demonstrate the relationship between

21. Babb, 74.

22. Guido of Arezzo, *Prologus in antiphonarium*, in Strunk, 213–14.

23. Guido, *Epistola de ignoto cantu*, in Strunk, 216.

24. For more on Guido’s methods, see Dolores Pesce, “Guido d’Arezzo, *Ut queant laxis*, and Musical Understanding,” in Russell E. Murray, Jr., Susan F. Wiess, and Cynthia J. Cyrus, *Music Education in the Middle Ages and the Renaissance* (Bloomington: Indiana University Press, 2010), 25–36.

mathematics and sound. It served as a touchstone for the theorist to confirm the Pythagorean dictum that all truth can be represented by numbers.

In the fourteenth century, a new style of obtaining a Pythagorean monochord division became popular. Previously, many manuscripts contained copies of Pseudo-Odo's division, wherein the notes are found in order, or Guido's fast division, which allows the student to mark the monochord quickly. Instead, the style of division used by *Ars Nova* theorists emphasizes even more the understanding of ratios.

Jean de Muris was primarily an astronomer and mathematician. His *Musica speculativa* (1323) discusses the relationship between number and sound by discussing the ratios of the consonances. He concludes the treatise with a monochord division that does not find the notes in order, but presents the ratios in order. In fact, he doesn't even bother with note names, but talks about points on the string by the letters a through v, and identifies intervals by their Greek names. Starting with the 2:1 ratio, he works through 3:2, 4:3, and finally 9:8:

If the chord a.b. is divided into two equal parts at point i., there will be a.i. to a.b. a diapason [octave]. Likewise, if a.i. is divided equally at q., there will be a.q. to a.b. a bisdiapason [double octave]. Therefore if anyone divides a.q. equally, there rises up the third octave.²⁵

After showing that any time you divide a string in half you get a note one octave higher, he moves on to the 3:2 ratio. Here he notes that by dividing the entire string in thirds you find a note (D) that is a diapente (fifth) away from the open string, but also a diatesseron (fourth) away from the G at the center of the string. Moving on to the 4:3 ratio, he notes that the note C is a diatesseron (fourth) away from Γ . Only after introducing these ratios does he move on to 9:8 and find A, which as we have seen is the first step in most divisions. From here he completes the gamut by repeating these four basic ratios.

Thus for Muris, the primary purpose of his monochord is not as a sight singing aid, but as a further way to demonstrate the mathematical principles involved in sound.²⁶ However, his division is rather tedious to actually create on a monochord, particularly when compared to Guido's fast method. In this way it is similar to the monochord division in the *Ars nova* treatise that has been attributed to Philippe de Vitry. Cecil Adkins speculates that, as these two treatises were widely copied and used, their lengthy divisions may have

25. Adkins, 151.

26. Interestingly, he also proposes a nineteen-stringed instrument, wherein each string corresponds by length to the string of the divided monochord, so it is possible to hear the intervals simultaneously.

contributed to the decline of the monochord as a useful instrument.²⁷ True, the cumbersome divisions would have made it increasingly frustrating for a student who wanted to use it in the traditional manner of a sight singing aid. However, the monochord was still useful as a mathematical tool, particularly as a tool to hear ratios as pitches. As such, it sometimes took a central role when theorists were exploring and writing about just tunings.

As a final example, we would like to consider the monochord division of Ramis de Pereia given in *Musica practica* (1482), which excited a great deal of fervor in its day. Like many others, Ramis recognized that the Pythagorean ditone (two intervals of 9:8, or 81:64, as is found from G to B) is much wider than a pure major third (5:4), sounding dissonant to the ear. Ramis designed a tuning that would include some pure thirds in addition to the pure fifths achieved in Pythagorean tuning, but as must happen, would have to have some non-pure fifths to compensate. The vehicle for presenting his new tuning was the monochord:

The regular monochord is accurately divided by Boethius with numbers and measurement. Although it is agreeable and useful for theorists, it is laborious and difficult for singers to understand. Truly, since we have promised to satisfy both [the theorists and the singers], we will render an extremely easy division of the regular monochord.²⁸

Thus Ramis's stated goal is to present a monochord division that is easy to execute as well as useful to practicing musicians and theorists. To do so was not easy, he says:

Let no one think that we came upon it with ordinary labor, inasmuch as we devised it with hard work during many sleepless nights, reading and rereading the precepts of the ancients and avoiding the error of the modern theorists. Anyone even moderately educated will be able to easily understand it.²⁹

What sets Ramis's approach apart? He sets the pitch of the monochord to A, which hearkens back to Boethius rather than the Γ that Pseudo-Odo introduced. He divides the string in half to find a, then divides the length from A to a in half to find D. This is actually no different than dividing the string in fourths to find D. However, the critical step follows: he divides D to a in half to find F (see **Figure 7**). This is in fact a fairly simple operation with dividers. Instead of swinging your dividers up to nine times to see how far off you are,

27. Adkins, 152–54.

28. Luanne Fose, "The *Musica practica* of Bartolomeo Ramos de Pareia: A Critical Translation and Commentary" (PhD dissertation, University of North Texas, 1992), 202.

29. *Ibid.*

then making adjustments, all you do is swing it twice. In doing this, however, he has created a pure minor third (6:5) between D and F, and a pure major third (5:4) between F and a. You can check it by counting the units from a to the end, four, and adding a fifth unit downward to find F, or 5:4. Adding one more equal unit creates the 6:5 of the pure minor third between D and F (see **Figure 8**).

Figure 7: Ramis finding a, D, and F.

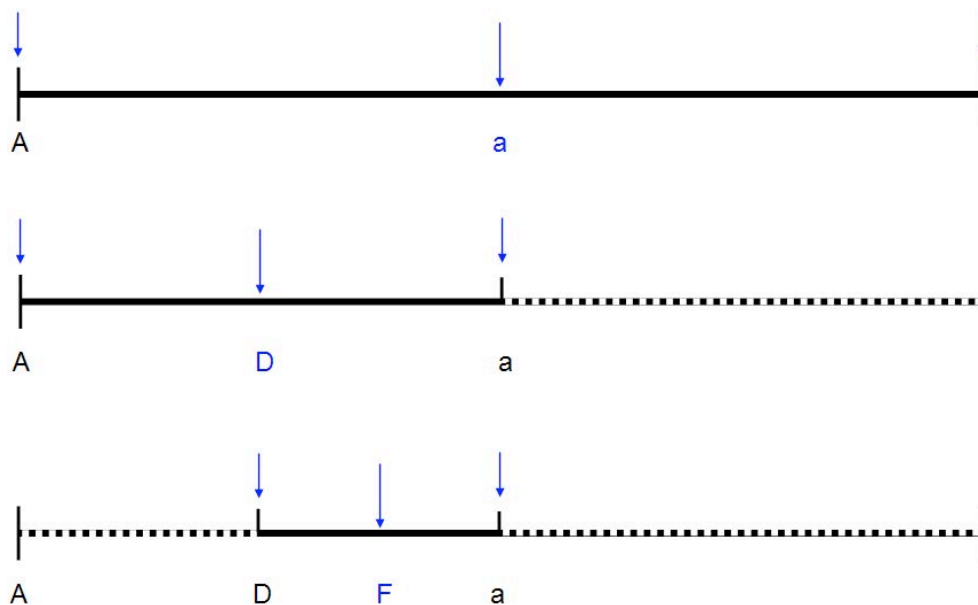
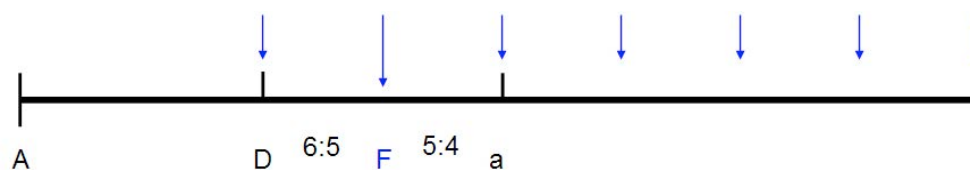


Figure 8: Pure thirds in Ramis's divisions.



Ramis then works around by pure fifths from A to find E and B, then finds a pure triad A-C-E, the same as D-F-A. This achieves some pure thirds and some pure fifths, but has an unusably tight fifth between G and D.

In his monochord division, Ramis is never explicit in his use of the pure thirds ratios, he just sneaks them in by doing what is admittedly an easier manual division. The result, of course, was radically different from the standard Pythagorean tuning. Ramis excited criticism for his “wrong” monochord division, as well as for proposing a new eight-syllable system to replace the Guidonian solmization hexachords.

Thus the monochord, central in medieval music theory, was the preeminent tool for exploring the relationship between number and sound. It was also a useful tool for sight singing, playing an important role in the introduction of the staff, solmization, and other early advances. It served theorists as an empirical tool, allowing students to get their hands on something physical and learn by doing. As such, it can do the same today.

II. The Monochord in the Modern Classroom

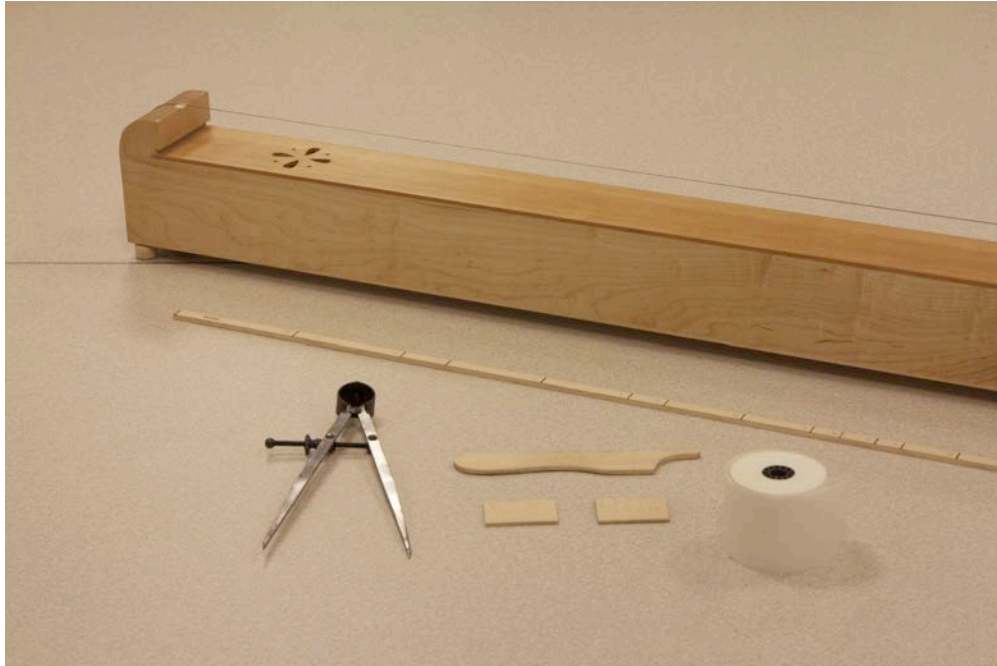
The monochord can be viewed primarily as a tool in making physically manipulable the numerical world that lies behind the musical one. It allows for the easy calculation and representation of various notes and intervals. It is, thus, a mathematical instrument on which different solutions can be tested, and the results of those solutions can be observed and weighed.

Much of the science embodied in the varied mathematical approaches to the monochord—or its conceptual foundations—is beyond the scope of a typical survey course, in no small part due to the complexity of the mathematical concepts. Few modern music students will be willing to deal with, for example, the 256:243 Pythagorean semitone. Besides, a full exploration of these mathematical concepts and their execution would demand far more time than can be given over to such an endeavor. Instead, we would like to propose a simple course module that highlights the basic theory and practice of the monochord, along with suggestions for more practical uses. While we have presented these lessons in rather ideal situations (an undergraduate course of about twenty students, a graduate course on the history of pedagogy for music education students, and at various conference workshops), we hope that they can also be adapted for larger classes and different situations.

Our goals for this lesson are fairly simple. The first is to introduce the concept of number and ratio as musical elements by demonstrating the simple derivation of intervals on the monochord. The second is to provide an experiential element to the students' understanding by guiding them through the construction of the Guidonian gamut using a simple method of division.

In addition to the monochord, the lesson needs simple dividers (one for each group of students), the completed division pre-marked on a stick (a more permanent version of the receipt paper), a set of movable bridges, and a roll of calculator tape or receipt paper (see **Figure 9**). In the absence of a monochord, a guitar with only one string can be used to demonstrate the basics of dividing the string, while the creation of the gamut can be done without the instrument.

Figure 9: From top: Monochord, Marked Stick, Divider, Bridges, and Calculator Tape.



A Primer on Sounding Number

We begin by reviewing with the students the basic information in their text about the medieval concept of number and sound, and of the central role of Pythagoras, Boethius, and ultimately Guido in the medieval speculative and practical traditions. In order to get them thinking about the physical properties, we ask them some simple questions in preparation for the session:

- How does one create an octave on a string?
- How does one create a fifth?
- How does one create the double octave?
- What are the principles involved in these processes?

To reinforce this, we use the simplest Pythagorean division scheme using only the ratios 2:1, 3:2, and 4:3. This can be done either by demonstration or by student involvement. The general placement is easy to do by eye, proving the intervals by playing the divided string:

- Using the monochord (tuned to a nominal G) and a movable bridge, demonstrate the 2:1 octave, then set a temporary bridge. Further divide that to create the double octave.
- Demonstrate 3:2 and 4:3 ratios to create D and C.
- To show the compounding and reciprocal quality of these intervals, set the D (3:2) with a temporary bridge, and then find the G (4:3) above that, comparing its placement with the octave G found in the first step.

This does two things. The first is that it makes physical the concept of number. And by having the students work through the problem, it clears up (at least for those who are not strong in math) the nature of the ratio—that it is best seen as a comparison of the part to the whole, not a true fraction or process of division.

Creating the Gamut

The next step is to have the students create the gamut using Guido's fast division, which we outlined earlier. The goal here is to get the students as involved as possible. To do this, set the monochord aside and prepare a paper "monochord" that the students in groups will use to mark the notes. For each group, prepare a tape long enough to have the entire string length of the monochord (or the guitar), with some room to spare. Using the pre-marked stick, mark the ends of the string, and then draw a line representing the string. Then, using a pre-marked stick, set the divider to one ninth of the string length, double-checking it by measuring the length of the string with the divider.

As we noted earlier, the choices made by each theorist about how to present his division can reveal how he intends the student to use the knowledge. This can equally be true for the modern teacher. The method that we use (see the Appendix for the class handout/overhead) is attractive for a couple of reasons, unrelated to Guido's use of the division. The first is that it doesn't start with a 2:1 or 3:2 division, so it immediately takes students out of their comfort zone by moving away from the ratios that they intuitively understand. The second is that it encourages for them to discuss various proofs of the intervals and to reinforce that there are multiple ways of deriving the same pitch using different ratios.

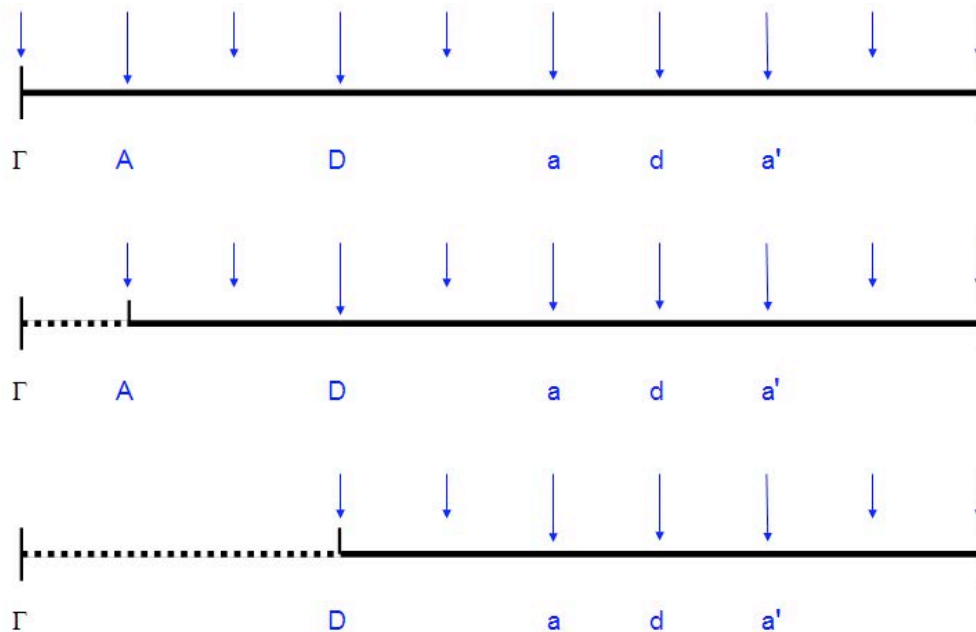
1. *Deriving pitch classes A and D.* The first division allows for the generation of pitch classes A and D. Set the dividers to one ninth of the entire string length. Starting from the bottom (left), mark A at 1/9, D at 3/9, a at 5/9, d at 6/9, and a' at 7/9. This creates a pure fifth between Γ and D (3:2) and a whole tone between Γ and A (9:8), along with their octaves and double octaves (see the first line of **Figure 10**). After the students have successfully made this division, you can explore some of the relationships behind it:

- The Γ -A second is marked at the 1st of 9 sections, leaving 8 sections, making a 9:8 ratio.
- The Γ -D fifth is marked at the 3rd of 9 sections, leaving 6 sections, making a 9:6 or 3:2 ratio.

After this is clear, some further observations can be made (looking at the second line of **Figure 10**). The A, since it was set at the first of 9 sections, contains 8 sections in itself, so:

- The octave a falls at the 4th of the 8 sections of A , proving the A - a octave ($4:2/2:1$).
- The D is marked at the 2nd of 8 sections, leaving 6 sections, proving the A - D fourth ($8:6/4:3$).

Figure 10: First division, for deriving pitch classes A and D .



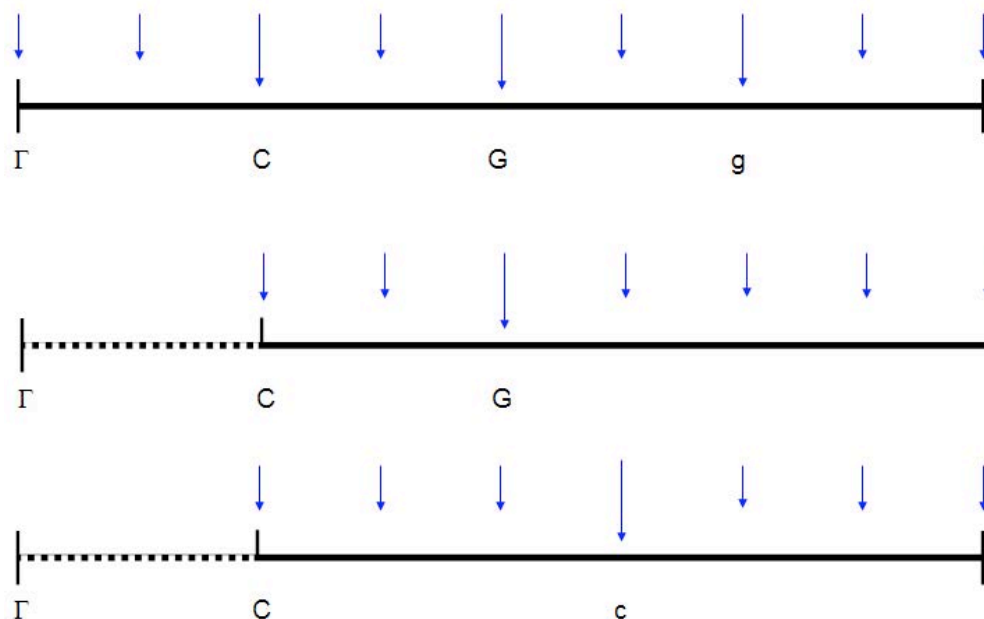
Finally, you can present them with a challenge: how can they prove the D to d octave? In order to do this, they will start on the D , noting that its length is divided in 6 parts, so that the small d falls on the third part, making a $2:1$ ratio (see the third line of **Figure 10**, supra).

2. *Deriving pitch classes B and E .* The second division replicates the first, starting on the note A . This yields the pitches B , E , b , e , and b' . The benefit of this for the student is that it allows him or her to redo the process and observe it leading a similar solution. The challenge is, of course, resetting the divider from one-ninth of the entire string length to one-ninth of the length from A to the end, and it is here that they learn a valuable lesson in the compounding of error—one that will resonate with them in a discussion of the Pythagorean comma as a result of consecutive perfect fifths. We often find that the “dead time” that this process creates allows the students to comment and ask questions that solidify earlier concepts or open the door to more in-depth discussion. It can, in fact, be a very creative period in the class.

3. *Deriving pitch classes C and G .* The third division, which yields pitch classes C and G , allows the students to deal with the $4:3$ ratio for the fourth. Go back to the entire length, and set the dividers to one eighth of the string

length. Starting from the bottom, mark C at $2/8$, G at $4/8$, and g at $6/8$. This creates a pure fourth between Γ and C, 4:3 (see **Figure 11**).

Figure 11: Third Division, for deriving pitch classes C and G.



The demonstrations here are straightforward:

- The Γ -C fourth is marked at the 2nd of the 8 sections, leaving 6 sections, and making an 8:6 or 4:3 ratio.
- The octave G is marked at the 4th of 8 sections, making a 2:1 ratio.

But it also presents two new challenges:

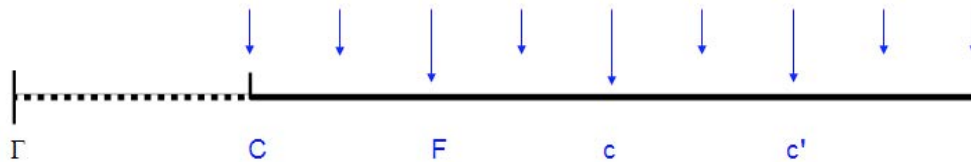
- How can one prove the C-g fifth?
- Where would one place the octave c?

The first is proven by noting that the C is divided into 6 parts, with the G lying on the second of these, creating a 6:4 or 3:2 ratio.

Likewise, starting with the C, the student will quickly see that the octave c falls on the third of the 6 divisions. They can mark this pitch, and prove it with the next division, which (as you can see from **Figure 12**) creates this note while deriving the F.

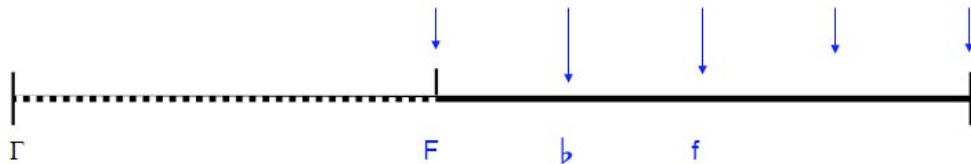
4. *Deriving pitch classes F and C.* For this division, start at the note C, and divide the remaining part of the string length into eight parts. Starting at C, mark F at $2/8$, c at $4/8$, and c' at $6/8$ (see **Figure 12**).

This is the same procedure as was used for the previous division, providing a 4:3 fourth and the octave and double octave of the C. The students will see that the c they marked in the last step matches the c created by the division. A few students will likely notice that the f could be placed at $5/8$, making

Figure 12: Fourth Division, for deriving pitch classes F and C.

a 2:1 octave with the F at $2/8$. As before, they can pencil it in and check it with the final division.

5. *Deriving pitch classes B \flat and F.* For the final division, start at F and divide the remaining length into four sections (see **Figure 13**). Mark $b\flat$ at $1/4$, f at $2/4$. This is the same procedure as the fourth division, creating another 4:3 fourth. But because the string length is shorter, we can set the dividers wide enough to mark fourths. Students will quickly see that the f they marked from the previous division matches the f derived here.

Figure 13: Fifth Division, for deriving pitch classes B \flat and F.

You now have a fully marked string with the complete gamut, with the exception of $b\flat$, d' and e' .³⁰ These can easily be derived, and the students can be challenged to find as many ways as possible to do this.

It is difficult to get through the entire division, with the attendant discussion, in one fifty-minute class period. If everything works logistically, it's not hard getting through the first three divisions, and that is probably enough to solidify the concepts. Then, like your favorite TV chef, you can pull out the finished product—a pre-marked stick against which they can compare the pitches they have marked.

III. Beyond the Gamut: Further Uses of the Monochord

Having created a gamut, there are several directions that you can move the discussion, depending on time and student inclination. In a survey class, you can use the monochord once again to discuss Guido's solmization, the hand, and learning chant. We have found that the last thing a student wants to talk

30. See note 20 above.

about in history class is theory. But pedagogy is a topic they are intimately familiar with—as students and as future teachers—and are naturally interested in. So using the instrument as a practical teaching tool has a strong resonance for the student, and it provides a strong context to help them understand the musical world of the past. By using a simple chant, students can work together to reinforce the basic intervallic components of the chant, or to teach individual phrases. Exploring this simple and mechanical method can provide insight into the daunting task of teaching and learning the chant repertoire. It also makes the improvements represented by Guido's solmization system real and apparent. From here it is easy to make the transition to talking about the Guidonian hand, and students can gain a great deal of insight by using the monochord and the hand in conjunction.

In terms of pure content, we can't say that this adds to what we already do in a class focused on the Guidonian system. If we are lecturing, we are essentially rehearsing everything covered here—that in the “bad old days” students learned chants by rote (and were often beaten when they failed), and that it was the same routine for each chant that they learned. We will go on to sing the praises of Guido and his marvelous invention and discuss how it revolutionized the process of learning and teaching. And we will also tell them that, in the end, the chants were still held in the huge repository of memory that the medieval students were building day by day. On the other hand, the advantage to having the monochord is that the students have a physical connection to the process of the pedagogy that was part of the daily lives of their counterparts of a thousand years ago. More powerfully, perhaps, than the use of the Guidonian hand, this allows the students to step into the role of either the medieval student or teacher.

Another approach is to continue with different types of divisions to explore issues of tuning and temperament. One of the most practical for many musicians is to compare a pure major third to an equal-tempered third and a Pythagorean ditone already found. Find the ditone by going up two 9:8 intervals from a starting point. Find the pure major third by dividing the length of string from that same starting point into five, and going up one swing of the dividers, creating a 5:4 ratio. Equal temperament cannot be derived with the same method, as it is based on an irrational number: the twelfth root of two. Instead, you must compute it by measuring the length of the string. Measure the length of string to your starting note, and divide this by 1.259921 to find the length of string one major third higher. You can also work up by half steps, dividing the length of the lower note by 1.059463 five times in a row. To hear the consonance of a pure major third on a single string, divide the string into nine parts and put a bridge at the fifth part (this is the same point as the pitch “a” in Guido's division used above). Now you have five parts on one

side of the string and four on the other, so comparing one side to the other will produce the 5:4 pure third.

Christian Meyer's *Mensura monochordi* provides an admirable text for exploring other divisions. The analytic tables at the back of the book condense each division into chart form, making them easy to browse and execute on the monochord. Of particular interest are Prosdocimo's fully chromatic Pythagorean division, in which the flats are higher than the sharps (Meyer, pp. 113 and 188), an anonymous division which takes you nearly all around the circle of fifths, making it easy to complete the loop and see the Pythagorean comma (Meyer, p. 145), just divisions (Meyer, pp. 224–30), and divisions which use the Greek enharmonic genera (Meyer, pp. 11 and 56 among others).³¹

As teachers, we are constantly faced with the demands of “engagement” and we think there is no better route to engagement than through the kind of identification that a student gains from working with something like the monochord—an engagement both with the concepts and with the actual practice. The speculative becomes a little less so, and the practical takes on practical meaning.

Appendix: Using Guido's Division of the Monochord³²

1. Deriving pitch classes A and D: Set your dividers to one ninth of the entire string length. Starting from the bottom (left), mark A at 1/9, D at 3/9, a at 5/9, d at 6/9, and a' at 7/9. You have created a pure fifth between Γ and D, 3:2, and a whole tone between Γ and A, 9:8.

- The Γ –A second is marked at the 1st of 9 sections, making a 9:8 ratio.
- The Γ –D fifth is marked at the 3rd of 9 sections, making a 9:6 or 3:2 ratio.

Note: The A that you created contains 8 sections, so:

- The octave a is marked at 4th of the 8 sections of A, proving the octave (2:1).
- The D is marked at the 2nd of 8 sections, proving the fourth (8:6/4:3)

How can you prove the D to d octave?

2. Deriving pitch classes B and E: Now take the distance from A to the end, and divide that into ninths. Starting from A, mark B at 1/9, E at 3/9, b at 5/9, e

31. These and other interesting divisions are also discussed in author Kate McWilliams' monochord blog: <http://www.monochordtheory.blogspot.com>.

32. The monochord is nominally tuned to G.

at $6/9$, and b' at $7/9$. You have created a pure fifth between A and E, 3:2, and a whole tone between A and B, 9:8.

- This is the same procedure as above, with a shorter string.

3. Deriving pitch classes C and G: Go back to the entire length, and set your dividers to one eighth of the string length. Starting from the bottom, mark C at $2/8$, G at $4/8$, and g at $6/8$. You have created a pure fourth between Γ and C, 4:3.

- The Γ -C fourth is marked at the 2nd of the 8 sections, making a 8:6 or 4:3 ratio.
- The octave G is marked at the 4th of 8 sections, making a 2:1 ratio.
- How can you prove the C-g fifth?
- Where would you place the c?

4. Deriving pitch classes F and C: Starting at C, divide the remaining part of the stick into eighths. Starting at C, mark F at $2/8$, c at $4/8$, and c' at $6/8$. You have created a pure fourth between C and F, 4:3.

- This is the same procedure as above, with a shorter string.
- Does your new c match the one from the previous division?

5. Deriving pitch classes $B\flat$ and F: Finally, starting at F, divide the stick into fourths. Mark $b\flat$ at $1/4$, and f at $2/4$. Now you have a pure fourth between F and $b\flat$.

- This is the same, using a division of 4 rather than of 8. So the F to $b\flat$ fourth is marked at the 1st of 4 parts, making a 4:3 ratio.
- The octave f is marked at the 2nd of 4 sections, making at 4:2 or 2:1 ratio.